

Cosmological models with gauge fields

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We discuss cosmological models involving homogeneous and isotropic Yang-Mills (YM) fields. Such models were proposed recently as an alternative to scalar models of cosmic acceleration. There exists a unique $SU(2)$ YM configuration (generalizable to larger gauge groups) whose energy-momentum tensor is homogeneous and isotropic in space. It is parameterized by a single scalar field with a quartic potential. In the case of the closed universe the coupled YM – doublet Higgs system admits homogeneous and isotropic configurations too. While pure Einstein-Yang-Mills (EYM) cosmology with the standard conformally invariant YM action gives rise to the hot universe, Einstein-Yang-Mills-Higgs (EYMH) cosmology has a variety of regimes which include inflationary stages, bounces, and exhibits global cycling behavior reminiscent of the Multiverse developed in time. We also discuss other mechanisms of conformal symmetry breaking such as string-inspired Born-Infeld (BI) modification of the YM action or field-theoretical quantum corrections.

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I. INTRODUCTION

With discovery of inflation as solution of the horizon and flatness problems in cosmology [1] it became widely accepted that, apart from gravity, some other homogeneous and isotropic fields have to be present at cosmological scale which mimic variable cosmological constant. Traditionally this role is attributed to a single scalar field, the inflaton, or several scalar fields [2]. Modern theories provide various candidates for inflaton varying from Higgs field of the standard model to more hypothetical moduli fields originating from compactified supergravity/string theory. Still, physical nature of the inflaton is far from being uniquely understood, and the choice of the inflaton potential remains mostly phenomenological. Similarly, popular current models of dark energy [3] involve scalar fields with rather exotic properties (quintessence, κ -essence, phantom) whose physical nature is far from clear. Moreover, no massless elementary scalar field was observed experimentally so far. Meanwhile, vector fields certainly do exist (and are massless before the spontaneous symmetry breaking) being basic ingredients of the Standard Model and its generalization. Therefore an idea to use vector fields instead or together with scalar ones to model inflation and dark energy seems to be appropriate. In fact, the model of inflation driven by vector field was suggested long ago [4], but remained unnoticed by cosmologists until recently when it was revived in the context of the dark energy problem [5]. Formation of YM condensates in superdense matter was discussed long ago by Linde [6].

There are two major reasons why vector fields were not welcome in cosmology, apart from their relative complexity. Spatially homogeneous configuration of a single (Abelian) vector field evidently can not be isotropic. Therefore, in order to fit the Friedmann-Robertson-Walker (FRW) cosmology one has to introduce (at least) a triplet of vector fields ensuring isotropy of the total energy-momentum tensor. Another problem is conformal invariance of the standard classical YM lagrangian, which leads within the FRW cosmology to the equation of state $w = P/\varepsilon = 1/3$, identical with that of the photon gas. Therefore, the solution for the coupled EYM system will be the hot Universe driven by the cold classical matter field [7]. Meanwhile, for an accelerated expansion one needs the equation of state $-1 \leq w \leq -1/3$, so the conformal invariance must be broken.

However, the first problem is avoided in the non-Abelian case: the $SU(2)$ triplet of YM fields admits an essentially non-Abelian configuration (with non-zero commutator of the matrix-valued potentials) whose stress-tensor exhibits three-dimensional homogeneity and isotropy. The second problem (conformal symmetry breaking) can be overcome passing to effective actions which account for quantum corrections either in the context of gauge theories or string theory. Various attempts to use YM vector fields in constructing dark energy models thus were undertaken during recent years [8].

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II. SU(2)-DRIVEN FRW COSMOLOGY

Consider the FRW interval in the conformal gauge

$$ds^2 = a^2(\eta) (d\eta^2 - d\vec{l}_k^2) \quad (1)$$

where $k = -1, 0, 1$ for open, flat and closed spatial geometry. As it was shown for $k = 1, 0$ by Cervero and Jacobs [9], Henneaux [10] and Hosotani [11] and for all k by Gal'tsov and Volkov [7], the following configuration

$$E_i^a = \dot{f} \delta_i^a, \quad B_i^a = (k - f^2) \delta_i^a, \quad (2)$$

parametrized by the single scalar function $f(\eta)$ of the conformal time gives rise to homogeneous and isotropic energy-momentum tensor. The YM lagrangian density in the conformal frame then reads

$$L = \frac{3}{8\pi a^4} (\dot{f}^2 - (k - f^2)^2) \quad (3)$$

so the electric part corresponds to the kinetic term, while the magnetic part — to the potential term in the action. The effective scalar field f is dimensionless (we use the units $\hbar = c = 1$), so in spite of similarity with the scalar field theory with the quatic potential, dependence of the lagrangian density on the scale factor a is different: the energy scales as $1/a^4$ which is characteristic for the conformal field. The energy-momentum tensor is traceless and the equation of state is

$$P = \varepsilon/3 \quad \text{with} \quad \varepsilon = \frac{3}{8\pi a^4} [\dot{f}^2 + (k - f^2)^2]. \quad (4)$$

Thus one obtains the hot Universe driven by non-thermal matter [7]: our classical YM configuration perfectly mimics the photon gas.

The YM equations reduce to equations of motion of a fictitious particle in the potential well

$$W_k = (k - f^2)^2 \quad (5)$$

which is especially interesting in the closed case $k = 1$ when it is the double well potential (Fig.1). Two minima correspond to neighboring topologically different vacua. We therefore observe that gravity lowers the potential barrier between topological sectors to a finite value, similarly to the Higgs field. Physically this similarity is due to attractive nature of both (contrary to repulsive nature of YM). If the energy is less than the height of the potential barrier, the particle oscillates around a single vacuum, when it is above the barrier, oscillations between different vacua are unsuppressed.

In the flat and open cases the potential has one minimum at $f = 0$ with $W = 0$ in the flat case $k = 0$ and $W = 1$ in the open case $k = -1$. so there are no topological effects.

Computing the Chern-Simons 3-form

$$\omega_3 = \frac{e^2}{8\pi^2} \text{Tr} \left(A \wedge dA - \frac{2ie}{3} A \wedge A \wedge A \right), \quad (6)$$

satisfying the equation

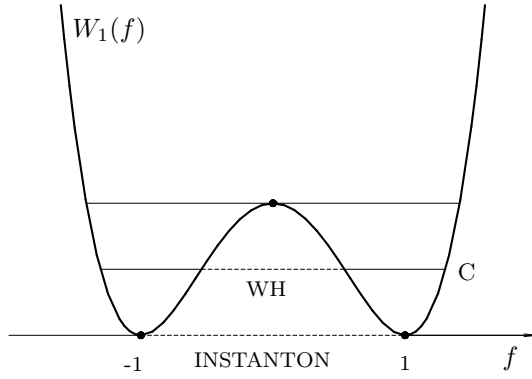
$$d\omega_3 = \frac{e^2}{8\pi^2} \text{Tr} F \wedge F, \quad (7)$$

one finds that it is non-trivial in the closed case $k = 1$, giving the winding number of the map $SU(2) \rightarrow S^3$:

$$N_{CS} = \int_{S^3} \omega_3 = \frac{1}{4} (f + 1)^2 (2 - f). \quad (8)$$

The vacuum $f = -1$ is topologically trivial: $N_{CS} = 0$, while the vacuum $f = 1$ is the next non-trivial one with $N_{CS} = 1$.

Generalization of the above ansatz for $SU(n)$ and $SO(n)$ gauge groups and further classical and quantum properties of EYM cosmological solutions were considered in a number of papers [12, 13] in the 90-ies. Behavior of small perturbations in cosmologies with vector fields was discussed more recently in [14].



A. Cosmological sphaleron

In the closed case there exists a particularly simple configuration $f = 0$ (f -particle sitting at the top of the barrier between two vacua) which in analogy with the sphaleron in the Weinberg-Salam (WS) theory was called the “cosmological sphaleron” [15]. It is worth noting that the localized particle-like EYM solutions similar to the WS sphalerons exist as well which are asymptotically flat regular particle-like solutions of the EYM equations discovered by Bartnik and McKinnon (for a review and further references see [16]). The repulsive YM stresses in these objects are compensated by gravity instead of the Higgs field in the WS sphalerons. Creation and decay of sphalerons generates a transition of the YM field between topological sectors, and it is accompanied by the fermion number non-conservation in presence of fermions [16]. The cosmological sphaleron has the topological charge $N_{CS} = 1/2$ like the sphaleron in the Weinberg-Salam theory.

The equation of motion of the f -particle

$$\ddot{f} = 2f(1 - f^2) \quad (9)$$

is solved indeed by $f = 0$, this correspond to the total energy

$$\dot{f}^2 + (1 - f^2)^2 = 1. \quad (10)$$

The YM field in this case is purely magnetic. A more general solution with the same energy describes rolling down of the f -particle (the sphaleron decay) [15]:

$$f = \frac{\sqrt{2}}{\cosh(\sqrt{2}\eta)}. \quad (11)$$

Rolling down to the vacuum $w = 1$ takes an infinite time, while the corresponding full cosmological evolution is given by

$$a = \sqrt{\frac{4\pi G}{g^2}} \sin \eta \quad (12)$$

and takes a finite time. Thus the cosmological sphaleron is quasi-stable. This conclusion is not modified if a positive cosmological constant is added.

B. Instantons and wormholes

An homogeneous and isotropic EYM system has interesting features also in the $k = 1$ space of Euclidean signature which is invoked in the path-integral formulation of quantum gravity. Actually, when $|f| < 1$, transitions between two topological sectors can be effected via underbarrier tunneling described by instanton and wormhole Euclidean solutions. In the Euclidean regime the first integral of the equations of motion reads:

$$\dot{f}^2 - (f^2 - 1)^2 = -C, \quad (13)$$

where C is an integration constant. Instanton corresponds to $C = 0$, it describes tunneling between the vacua $f = \pm 1$. It is a self-dual Euclidean YM configuration for which the stress tensor is zero, therefore a conformally flat gravitational field can be added just as a background.

Tunneling solutions at higher excitation levels $0 < C \leq 1$ are not self-dual. In flat space-time they are known as a meron ($C = 1$, the Euclidean counterpart of the cosmological sphaleron, $N_{CS} = 1/2$) and nested merons $0 < C < 1$, $1/2 < N_{CS} < 1$. The energy-momentum tensor of the meron is non-zero, and in flat space this solution is singular. When gravity is added, the singularity at the location of a meron expands to a wormhole throat, and consequently, the Euclidean topology of the space-time transforms to that of a wormhole. Topological charge of the meron wormholes is zero, the charge of the meron being swallowed by the wormhole [17]. The total action of these wormholes diverges because of slow fall-off of the meron field at infinity, so the amplitude of creation of baby universes associated with the Euclidean wormholes is zero. However, when a positive cosmological constant is added (inflation) the action becomes finite due to compactness of the space. Such solutions can be interpreted as describing tunneling between the de Sitter space and the hot FRW universe.

Adding the positive cosmological constant, we will get similar first integrals both for the f -particle and the cosmological radius:

$$\dot{f}^2 - (f^2 - 1)^2 = -C, \quad \dot{a}^2 + (\Lambda a^4/3 - a^2) = -C/(e^2 m_{Pl}^2). \quad (14)$$

Solutions describe independent tunneling of f and a with different periods T_f , T_a depending on the excitation level C . To be wormholes, they must obey a quantization condition $n_f T_f = n_a T_a$ [18] with two integers. However, for a specific value of the cosmological constant

$$\Lambda = \frac{3}{4} m_{Pl}^2, \quad (15)$$

it was found [19] that $T_a = T_f$ for all $C \in [0, 1]$. In particular, for $C = 1$ (the meron limit) the radius a becomes constant (Euclidean static Einstein Universe). For $C \neq 1, 0$ the solutions describe creation of the baby universes (which was invoked in the Coleman's idea of the "Big fix"). Remarkably, under the above conditions, the total action (gravitational plus YM) is precisely zero [19]:

$$S_{YM} + S_{gr} = 0. \quad (16)$$

Thus, the pinching off of baby universes occurs with unit probability. For later work on EYM wormholes see [20].

III. EINSTEIN-YANG-MILLS-HIGGS COSMOLOGY

Consider now the EYMH action with complex doublet Higgs:

$$S = \int \left\{ -\frac{1}{16\pi G} R - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} (D_\mu \Phi)^\dagger D_\mu \Phi - \frac{\lambda}{4} (\Phi^\dagger \Phi - v^2)^2 \right\} \sqrt{-g} d^4 x, \quad (17)$$

where

$$D_\mu \Phi = \partial_\mu \Phi + e A_\mu^a T_a \Phi. \quad (18)$$

It is easy to see that in the case of spatially closed FRW cosmology an ansatz for Higgs

$$\Phi = h(t) e^{i\xi(t)} U \Phi_0, \quad U = e^{2\chi T_a n^a}, \quad \Phi_0^\dagger \Phi_0 = 1, \quad (19)$$

is compatible with homogeneity and isotropy of the full EYMH system. Indeed, with the YM parametrization

$$e A_\mu^a T_a = \frac{1 - f(t)}{2} U \partial_\mu U^{-1}, \quad (20)$$

the covariant derivative reduces to the ordinary one

$$D_\mu \Phi = \frac{1 + f}{2} \partial_\mu \Phi. \quad (21)$$

Thus an homogeneous and isotropic cosmology does exist for Higgs in the fundamental representation. On the contrary, the triplet Higgs leads to anisotropic cosmology [21].

The Higgs phase rotation factor $e^{i\xi(t)}$, which is compatible with the desired symmetry, in the flat space leads to infinite energy and must be omitted, but for the closed FRW cosmology its contribution is finite, so we may keep it.

The EYM action contains three different mass parameters: the Planck mass, the mass of the W-boson, and the Higgs mass

$$M_{\text{Pl}} = \frac{1}{\sqrt{G}}, \quad M_W = ev, \quad M_H = \sqrt{\lambda}v. \quad (22)$$

We then rescale the Higgs function $h \rightarrow hM_{\text{Pl}}$ and introduce the dimensionless parameters

$$\alpha = \frac{M_W}{eM_{\text{Pl}}} = \frac{v}{M_{\text{Pl}}}, \quad \beta = \frac{M_H}{M_W} = \frac{\sqrt{\lambda}}{e}. \quad (23)$$

Finally, keeping in mind that the pure EYM system has a natural lengths scale $l = 1/(eM_{\text{Pl}})$, we present the metric in terms of the dimensionless lapse and scale functions N, a :

$$ds^2 = l^2 \left\{ -N^2 dt^2 + a^2 \left[d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2) \right] \right\}. \quad (24)$$

Then substituting the ansatz (19,20) and integrating over the three-sphere we obtain the one-dimensional reduced action

$$S_1 = \int \left[\frac{3}{8\pi} \left(aN - \frac{\dot{a}^2 a}{N} \right) + \frac{3f^2 a}{2N} + \frac{\dot{h}^2 a^3}{2N} + \frac{h^2 \dot{\xi}^2 a^3}{2N} - V_f - V_h - V_{\text{int}} \right] dt \quad (25)$$

$$V_f = \frac{3N(f^2 - 1)^2}{2a}, \quad V_h = \frac{\beta^2}{4}(h^2 - \alpha^2)^2 Na^3, \quad V_{\text{int}} = \frac{3}{4}h^2(f+1)^2 Na \quad (26)$$

where we have omitted an overall factor $\pi/4$ and the total derivative in the scalar curvature term.

The equation for the Higgs phase rotation variable ξ

$$\frac{d}{dt} \left(\frac{h^2 a^3 \dot{\xi}}{N} \right) = 0 \quad (27)$$

can be integrated

$$\dot{\xi} = \frac{\sqrt{2}jN}{h^2 a^3}, \quad (28)$$

where j is the integration constant. Then it is easy to check that the remaining dynamics can be derived from the action in which the ξ -kinetic term is replaced by the potential term

$$V_j = \frac{j^2}{Nf^2 a^3}. \quad (29)$$

Variation with respect to N leads to the constraint equation

$$a(\dot{a}^2 + 1) = \frac{8\pi}{3}\varepsilon, \quad (30)$$

and it is convenient to fix the gauge $N = 1$ afterwards. For the energy density we then get

$$\varepsilon = T_f + T_h + V_f + V_h + V_{\text{int}} + V_j, \quad (31)$$

where the first two terms are kinetic:

$$T_h = \frac{1}{2}\dot{h}^2 a^3, \quad T_f = \frac{3}{2}\dot{f}^2, \quad (32)$$

and the remaining are four potentials:

$$V_f = \frac{3(f^2 - 1)^2}{2a}, \quad V_h = \frac{\beta^2}{4}(h^2 - \alpha^2)^2 a^3, \quad V_{\text{int}} = \frac{3}{4}h^2(f+1)^2 a, \quad V_j = \frac{j^2}{f^2 a^3}, \quad (33)$$

The acceleration equation is obtained by variation of the action with respect to a with account for the constraint:

$$a^2\ddot{a} = -\frac{4\pi}{3}(\varepsilon + 3p), \quad (34)$$

leading to the following expression for pressure:

$$p = T_h - V_h + V_j + \frac{T_f}{3} + \frac{V_f}{3} - \frac{V_{\text{int}}}{3}, \quad (35)$$

and therefore,

$$a^2\ddot{a} = -\frac{8\pi}{3}(2T_h + 2V_j + T_f - V_h + V_f). \quad (36)$$

The field equations for the YM and Higgs scalar functions read

$$\begin{aligned} \ddot{f} + 3\frac{\dot{a}}{a}\dot{f} &= -\frac{3}{2a^2}h(f+1)^2 - \beta^2(h^2 - \alpha^2)h + \frac{2j^2}{h^3a^6}, \\ \ddot{f} + \frac{\dot{a}}{a}\dot{f} &= -\frac{3}{2}h^2(f+1) - \frac{6}{a^2}(f^2 - 1)f. \end{aligned} \quad (37)$$

From the acceleration equation is clear that kinetic terms always produce deceleration, while the potential terms are partly accelerating. In the transient regimes when different potential terms dominate, one has the following equations of state:

Dominant potential	$w = p/\varepsilon$	type
Higgs V_h	$w = -1$	cosmological constant
Higgs phase rotation V_j	$w = 1$	stiff matter
YM potential V_f	$w = 1/3$	radiation
Interaction term V_{int}	$w = -1/3$	string gas

A. Standard model scale

Behavior of solutions essentially depends on the parameters α , β . Consider first the scales relevant to the Standard model. In this case $\alpha \sim 10^{-17}$ and β is of the order of unity. The corresponding dynamical scale is far from the Planck scale and matter contribution to the action is of the order of $\alpha^4\beta^2$. The corresponding values of the scale factor must be of the order $1/\alpha^2\beta$. Then the main contribution to evolution of the scale factor will come from the Higgs potential V_h . Typical regimes of the evolution of the Higgs scalar correspond to motion near the extremal points of the potential: the minima $|h| = \alpha$ and the local maximum $h = 0$. In the first case one observes small oscillations, in the second – slow rolling down from the top of the potential barrier. While the frequency of oscillations is proportional to β , the rolling down velocity is much less, namely of the order of $\alpha^2\beta$. A substantial variation of the scale factor will correspond to time intervals of the order of $1/\alpha^2\beta$, so the cosmic acceleration will be proportional to the average in time values of potential and kinetic energy of the scalar field $\langle V_h - 2T_h \rangle$. For harmonic oscillations $\langle T_h \rangle = \langle V_h \rangle$, so the acceleration will be negative. The rolling down regime is exponential, so the characteristic time T_{roll} will depend on initial conditions as $-\ln h(0)$. During the rolling time the scale factor will exponentially grow up with the Hubble parameter proportional to $\alpha^2\beta$. Thus an exponential expansion will be insignificant unless we choose $-\ln h(0) \gg 1/\alpha^2\beta$.

It is known that in inflationary models with power-law potential the oscillating universe regime is possible when during contraction phase the amplitude of scalar field oscillations grows up and as a result the scalar field climbs close to the potential top. Then the rolling down regime follows with the corresponding expansion of the scale factor. When rolling down terminates, the oscillations starts again and one enters a new cycle. But with the YM field present, the probability to hit the relatively small region of the phase space for triggering the rolling-down regime decreases, since the cumulating chaotic oscillations and interaction with the YM component deviate substantially the phase trajectory from reproducing precisely the previous cycle. Therefore for small α the cycling behavior demands fine tuning of the initial data for realization of the cycling universe. Otherwise, the evolution will terminate with a collapse to a point.

As about the YM component, its amplitude will oscillate with a period inversly proportional to the average value of the scalar field $\langle h \rangle$ (i.e. of the order α^{-1}) around the minimum $f = -1$, since oscillations will be governed by the interaction potential V_{int} , while the YM potential V_f will be negligibly small. In the case when the scalar field sits

at the top of the potential barrier (what is possible if $V_j = 0$), the YM field will oscillate in the potential V_f . The oscillation period will then be proportional to the value of the scale factor, but for sufficiently fast increase of a the frequency becomes imaginary, describing an exponential relaxation of the YM field to the vacuum value. The scale factor itself in this special case will be either exponentially growing, or collapsing depending on initial conditions. This can be shown analytically. Indeed, for $h = 0$ the non-zero potential energy of the Higgs field will play the role of the cosmological constant equal to $\Lambda = \alpha^4 \beta^2 / 4$. As a result, independently of the dynamics of the YM field the equation of state will be

$$p = \frac{\varepsilon - 4\Lambda a^3}{3}. \quad (38)$$

Substituting this into the continuity equation

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}p = 0, \quad (39)$$

after trivial integration we obtain

$$\varepsilon = \Lambda a^3 + \frac{C^2}{a}, \quad (40)$$

where C - is the integration constant. For instance, for the sphaleron configuration $f = h = 0$ with no field dynamics the energy density is given by

$$\varepsilon = V_h + V_f = \frac{\Lambda a^3}{4} + \frac{3}{2a}, \quad (41)$$

which results in $C^2 = 3/2$. Using the value (40) for the energy density, we can solve the constraint equation for the scale factor:

$$\dot{a}^2 + 1 = \gamma^2 a^2 / 4 + 4\eta^2 / a^2, \quad \text{where } \gamma \equiv 2\alpha^2 \beta \sqrt{\frac{2\pi}{3}}, \quad \eta \equiv C \sqrt{\frac{2\pi}{3}}. \quad (42)$$

A simple solution arises when the right hand side of this equation is a full square. For this to happen, one has to impose the following relation between the parameters:

$$\gamma\eta = 1. \quad (43)$$

Taking the square root of the constraint equation, one obtains

$$\dot{a} = \gamma a / 2 - 2 / (\gamma a). \quad (44)$$

Then the solution for the scale factor will read

$$a = \sqrt{\frac{2}{\gamma^2} + \left(a_0^2 - \frac{2}{\gamma^2}\right) e^{\gamma t}}, \quad (45)$$

где $a(0) = a_0$. Note that the constraint equation is invariant under the time reflection, so if $a(t)$ is a solution, then $a(-t)$ will also be a solution. So for brevity we do not write \pm in the Eq. (44) as well as in the solutions for $a(t)$.

Depending on the values of the parameters, one observes either cosmological singularity, or inflation. For $a_0 = 2/\gamma$ the solution will be the static universe $a(t) = a_0$: in this case the negative pressure of the scalar field is exactly compensated by the positive contribution of the YM field.

If the right hand side of (42) is not a full square (which can be expected in our Standard model scales case since the relations (43) does not hold in view of $\alpha \ll 1$), then an analytic formula for the time dependence of the scale factor is more complicated:

$$a = \sqrt{\frac{e^{-\gamma t}}{\gamma^4 \hat{C}} (1 - \gamma^2 \eta^2) + \frac{2}{\gamma^2} + \hat{C} e^{\gamma t}}. \quad (46)$$

Here the integration constant was introduced as \hat{C} and not as $a(0)$. Evidently, for small α the solution will be a growing exponent with the power coefficient proportional to α^2 . Coming back to dimensionful time parameter will get an exponential of $\alpha^2 / T_{Pl} \sim 10^9 \text{sec}^{-1}$. Comparing this with an observed Hubble constant 10^{-17}sec^{-1} we get the needed value of the mass of the scalar field $10^{-30} M_{Pl} = 10^{-2} \text{eV}/c^2$.

B. Planck scale

Now consider the case of α and β of the order of unity, so all the quantities are of the Planck scale. While in the case $\alpha^2\beta \ll 1$ the behavior of the system was determined by the fixed points of the Higgs potential, now we have to find the fixed points of full system of equations (37) and (36).

Equating to zero the time derivatives of the variables we get the following system :

$$\begin{aligned}\frac{3h}{2a^2}(f+1)^2 + h\beta^2(h^2 - \alpha^2) &= 0, \\ \frac{3h^2}{2}(f+1) + \frac{6f}{a^2}(f^2 - 1) &= 0, \\ \frac{a\beta^2}{4}(h^2 - \alpha^2)^2 - \frac{3}{2a^3}(f^2 - 1)^2 &= 0.\end{aligned}\tag{47}$$

Denoting the variables collectively $Q = \{h, f, a\}$ we will get a solution depending on parameters $Q = Q_0(\alpha, \beta)$. This solution has to satisfy the constraint equation (30):

$$a_0(\alpha, \beta) = \frac{8\pi}{3}\varepsilon(\alpha, \beta).\tag{48}$$

Not every solution of (47) does it: e.g. the vacuum state $h = \pm\alpha$, $f = -1$ does not. As a result, the physical fixed points of the system of equations are realised in the parameter space only on the curve given by the Eq. (48).

Obviously, the solution $f = h = 0$ satisfies the first two equations of the system (47), while from the third equation we find the complete solution:

$$h = 0, \quad f = 0, \quad \alpha a = 2/\sqrt{\tilde{\beta}}.\tag{49}$$

He we denoted $\tilde{\beta} = \beta\sqrt{8/3}$ to simplify further relations. This static solution was already found in the previous section as $a_0 = 2/\gamma$.

Finally, the last non-trivial solution of the system (47) reads:

$$\alpha h^2 = f = \alpha \frac{\tilde{\beta} - 1}{\tilde{\beta} + 1}, \quad \alpha a = \frac{2\sqrt{2}}{\sqrt{\tilde{\beta} + 1}}.\tag{50}$$

In the limit $\tilde{\beta} \rightarrow 1 + 0$ this solution coincides with the previous one. For $\tilde{\beta} < 1$ the solution of the type (50) does not exist. One can notice that the parameter α is just the scale one.

Let us try to give qualitative description of the fixed point corresponding to the solutions described above. For the first solution we have the following configuration. The field h sits at the local maximum of its potential V_h : $h_0 = 0$. This switches off the interaction between the scalar and the YM components: $V_{\text{int}} = 0$. Correspondingly, the fixed points of the YM field will be extrema of the potential V_f . These are ± 1 , 0, from which only $f_0 = 0$ satisfies the constraint equation.

In the second case the fixed point for h will be not the maximum, but the minimum of the potential. This is not the minimum $h = \pm\alpha$, since interaction with the YM field shifts the point of minimum in such a way that $0 < h_0 < \alpha$. Similarly, for the YM field the extremal points 0, 1 turn out to be shifted: the local maximum to the right, and the minimum to the left according to the relation $f_{\text{max/min}} = (1 \pm \sqrt{1 - a^2 h^2})/2$. From the constraint equation we get that for $l < 3$, the fixed point is the local maximum; for $l = 3$ the maximum and the minimum coincide forming the inflection point; with further increasing l the fixed point will be the local minimum.

Finally, the constraint equation shows in which region of the phase space the system can be depending on parameters, or, conversely, which should be the energy of the scalar field for the desired regime of evolution associated with the motion near one or another fixed points. Therefore for two families of solutions (49) and (50), respectively, we find the following relations:

$$\alpha_1 = \frac{1}{\sqrt{2\pi\tilde{\beta}}}, \quad \alpha_2 = \frac{\tilde{\beta} + 1}{\tilde{\beta}\sqrt{8\pi}}.\tag{51}$$

To describe evolution of the system in the vicinity of fixed points one has to linearize the equations of motion around them. Excluding the momentum variables, we obtain the matrix equation:

$$\delta\ddot{Q} = M\delta Q,\tag{52}$$

where the matrix M is obtained by differentiation of the equations of motion and the Friedmann equations over $Q = \{h, f, a\}$ and substituting the values of the variables in the fixed point. At the same time we have to take into account the dependence between the parameters imposed by the constraint equation. Then we obtain for $M(l)$:

$$M_1 = \frac{1}{4\pi} \begin{pmatrix} \frac{3}{4}(\tilde{\beta} - 1) & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (53)$$

while in the second case:

$$M_2 = \begin{pmatrix} \frac{3(\tilde{\beta}^2 - 1)}{32\pi} & -\frac{3\sqrt{2}(\tilde{\beta} + 1)^{5/2}}{128\pi^{3/2}\tilde{\beta}^2} & \frac{3\sqrt{2}(\tilde{\beta} + 1)^3\sqrt{\tilde{\beta} - 1}}{512\pi^2\tilde{\beta}^2} \\ \frac{3\sqrt{\tilde{\beta} - 1}}{\sqrt{2\pi}(\tilde{\beta} + 1)} & -\frac{3(\tilde{\beta} - 3)(\tilde{\beta} + 1)}{16\pi\tilde{\beta}} & -\frac{3(\tilde{\beta} + 1)^{3/2}(\tilde{\beta} - 1)}{32\pi^{3/2}\tilde{\beta}^2} \\ \sqrt{\frac{\tilde{\beta} - 1}{2}} & \frac{(\tilde{\beta} + 1)^{3/2}(\tilde{\beta} - 1)}{8\sqrt{\pi}\tilde{\beta}^2} & \frac{(\tilde{\beta} + 1)^2}{16\pi\tilde{\beta}^2} \end{pmatrix} \quad (54)$$

It is easy to explore the case $\tilde{\beta} < 1$, when only the matrix M_1 is relevant. For the scale factor the fixed point will be the focus by h and nodes by f and a describing an exponential expansion or contraction of the universe. For this we obtain the following critical value of the scale factor $a_0 = \sqrt{8\pi}$, when the inflationary potential V_h is compensated by the deflationary potential V_f in the equation.

In order to determine the system behavior in the general case, one has to find the eigenvalues of the linear system.

$$\det(M_{1,2} - \mu^2 \mathbb{I}) = 0. \quad (55)$$

where the eigenvalues μ of the full system in the six-dimensional phase space enter squared into the three-dimensional system which is left after eliminating momenta.

The matrix M_1 is diagonal, so that the eigenvalues are obvious. The eigenvalue for the scalar field is $\mu_h^2 = 3/4(\tilde{\beta} - 1)$. For $\tilde{\beta} > 1$ we have a node. For $\tilde{\beta} < 1$, the eigenvalues become pure imaginary, that is the node is transformed to focus. This means that the quadratic in h interaction potential V_{int} dominates over the quadratic term of the Higgs potential, so the effective potential at the point $h = 0$ has a minimum, but not a local maximum. Eigenvalues of the gauge field and the scale factor are obviously always real. On the phase portrait this fixed point is a node, and one therefore has expansion or contraction of the universe. Correspondingly, the system is unstable in the vicinity of fixed point parameterized by the equations (49).

Eigenvalue of the matrix M_2 can also be found analytically since the corresponding equation (55) is cubic in terms of μ^2 . Their explicit form, however, is too long, so we do not present it here. One root is always real, as it should be for the cubic equation. It is negative and the corresponding fixed point is focus. Two other roots are complex on the interval $\tilde{\beta} \in [1.8, 7.5]$. Their real parts are equal, while imaginary parts differ in sign describing different orientation (left/right handed) of phase trajectories. For $1 < \tilde{\beta} < 1.8$ the roots are real and positive, while for $\tilde{\beta} > 7.5$ — negative. In the first case one has nodes, in the second — elliptic orbits. Thus, on the interval $\tilde{\beta} > 7.5$ the fixed points parameterized by (50), are focal points in the full six-dimensional phase space, so the system is stable in the vicinity of these points.

From this analysis it follows that one of the main features of the behavior of the system on Planck scale is that apart from unstable solution of the type of static universe, when small deviation trigger long cycles of expansion or contraction, there are quasi-static regimes characterized by dynamical equilibrium between the scale factor and matter, which are stable against small perturbations.

Finally, we discuss the role of the Higgs phase rotation parameter j . When $j \neq 0$, one observes the regimes typical for the second family of fixed points (while the first family can not be realized since now $h \neq 0$). This additional parameter can be used to set the system into the state close to stationary points. Without it, we have some prescribed dependence between the other parameters, $\alpha(\beta)$, for every fixed point, which may not be satisfied for the particular theory chosen. With $j \neq 0$, one has more flexibility in the parameter space to impose the desired evolution regime of the system.

C. Numerical experiments

Numerical solutions confirms the above qualitative considerations. We are particularly interested in illustrating the system behavior in different regimes described in the previous subsection. On Figs. (1–2) we present the solution

exhibiting the existence of quasi-stable state. The system enters this state with negative and leaves it with positive cosmic acceleration, with concrete values depending on initial conditions. The Fig. (3) illustrates transition from the quasi-stable local minimum near zero to a true minimum $h = \alpha$, after inflationary expansion of the scale factor. These figures show behavior of the system with Planck scale parameters, when interaction with the YM field is sufficiently strong to change substantially vacuum state of the scalar field.

The cyclic evolution is presented on Fig. (4). For this solution we have chosen the parameter values such that the periods of the expansion cycles are comparable to the period of the oscillations of the scalar field. In this case interaction between scalar and YM fields is not strong, but still the YM field plays crucial role at the moments of bounces. Since the oscillation frequencies of h and f are different, the value of the YM field at the next bounce may differ significantly for that at the previous bounce. With high probability the subsequent cycles differ from each other with the YM field values being chaotically distributed at the moments of bounces. This is the main difference of the large time scale behavior of the EYMH system as compared with the pure scalar cosmology.

Finally, on Fig. (5) we illustrate the dynamics of the EYMH universe for asymptotically small value $\alpha = 10^{-6}$. When the oscillation amplitude of the scalar field is relatively big (for this figure $h(0) = 10^{-2}$), the potential term exceeds the kinetic term leading to weakly accelerating regime. But with growing scale factor the scalar field amplitude decreases and expansion converts to contraction.

D. EYMH hybrid inflation

As was described in previous subsection, the EYMH system demonstrates a wide variety of behavior in different regimes. But the most interesting among them seems to be the next one. The EYMH system may be considered as a very natural variant of the hybrid inflation, introduced by Linde [22]. The main idea of the hybrid inflation was to add a massive scalar field f to the Higgs field h so that the total potential reads

$$V(f, h) = \frac{m^2}{2} f^2 + \frac{\lambda'}{2} f^2 h^2 + \frac{\lambda}{4} (h^2 - v^2)^2. \quad (56)$$

In this case inflation consists of two stages. When f is larger then the critical value $f_c = \lambda v^2 / \lambda'$, the Higgs field is trapped on the top of its potential due to the interaction between the scalar fields. During this stage the field f slowly rolls under f_c and then the second stage starts — the roll of h . Inflation ends when h reaches its true minimum $|v|$.

In our system there is a similar trapping potential V_{int} . Mention that it does not enter the Friedmann equation for acceleration, therefore the inflationary potential will be only V_h . Next, V_{int} depends on the gauge field and the scale factor. The gauge field changes slowly (especially if one sets it in the minimum $f = 1$ of its potential V_f), but the scale factor in the denominator grows exponentially, therefore the first stage when h is trapped will be rather short. But during the first stage the scalar field will be lifted closer to the top of the potential V_h , what implies a prolonged second stage of inflation with the slow roll of the Higgs field. In other words, large inflation occurs when the initial value of the Higgs field, h_0 , is strictly chosen to be in vicinity of the top: $h_0 \simeq 0$. But this condition can be weakened significantly due to the trapping potential which will automatically prepare the Higgs field in the right position before the rolling down.

As was mentioned above, the gauge field plays crucial role in the trapping of the Higgs field: it switches the trapping on, when $f_0 = 1$, and off, when $f_0 = -1$. In the last case the Higgs field will fall freely, reproducing the standard ‘slow roll’ scenario. The initial value of the scale factor a_0 affects the duration of the trapping: when a reaches the critical value $a_c \simeq \sqrt{6}/(\alpha\beta)$ (what happens rather fast due to the exponential growth of a), the trapping will end.

Now let us turn to the numerical calculation of the number N of e -folds during this hybrid inflation. On the Fig. (6) one can see the dependence of the N on the initial value h_0 of the Higgs field. There are three plots: N_- , when trapping is switched off by the gauge field $f_0 = -1$; N_+ , which was calculated for the trapping with $f_0 = 1$ and $a_0/a_c = 1/5$; their ratio N_+/N_- . The values N_{\pm} are normalized by the number $N = 60$ of e -folds during the real inflation of our universe [23]. One can see that even for rather weak trapping with a_0 being just five times smaller than the critical value, there is a 20 – 40% gain of e -folds. This gain grows with the increase of h_0 , as expected, because the lift of h to the top of V_h is significant when h_0 is far from the top. Also the needed number of e -folds $N \sim 60$ can be obtained in twice wider range of h_0 when the trapping is on.

So switching on of the trapping due to the interaction with the gauge field may really enhance inflation. On the Fig. (7) we can see how this amplification will depend on the ratio $\ln(a_0/a_c)$. For chosen parameters the absence of trapping when $a_0 \gg a_c$ gives us about thirteen e -folds. But the choice of several e -folds smaller a_0 (so that $a_0 \ll a_c$) will allow us to gain a large total number of e -folds. Actually there is even a divergence in the e -folds number when the trapping potential drives the scalar field exactly on the top of the potential V_h with infinite inflation. Of cause this trapping scenario is significant when $j = 0$, and the number of e -folds greatly decreases with the growth of j .

We conclude this section with the following remarks. Coupled Yang-Mills-Higgs dynamics for closed FRW universe gives rise to new interesting evolution types which include transient regimes of cosmic acceleration, bounces and cyclic evolution. Presence of the YM component and Higgs phase rotation parameter changes substantially the dynamics of the universe at small scale factors. The interaction with the gauge field holds the Higgs field near the top of the potential, and the balance of accelerating and decelerating potentials of scalar and vector fields can freeze the scale factor. The phase rotation acts in reverse, making the dynamic of the system to be more similar to the scalar field with power-low potential, but its kinetic nature leads to the opposite sign in the potential at very small distances. Also it increases the number of free parameters, which can be useful in quantitative analysis.

One intriguing feature is possibility of an infinite sequence of cycles whose parameters change chaotically due to evolution of the YM component. This resembles the Multiverse models [24] realized as sequence of universes with different parameters in the ultralarge time scale.

The system of interacting Higgs and YM fields can be considered as a very natural candidate for the hybrid inflation scenario, which is richer than a standard slow roll inflation with a scalar field. The particular feature of the model is that due to the vector nature of the gauge field, whose energy density depends on the scale factor, the EYMH inflation also inherits this dependence. There can be a large inflation in a small universe, and a small inflation in a large universe. This looks quite similar to the current views on the evolution of the universe with large initial inflation and slow late-time acceleration.

IV. NON-ABELIAN BORN-INFELD (NBI)

Open string theory suggests the following generalization of the Maxwell Lagrangian (applicable to any dimensions):

$$L = \frac{\beta^2}{4\pi} \left(\sqrt{-\det(g_{\mu\nu} + F_{\mu\nu}/\beta)} - \sqrt{-g} \right), \quad (57)$$

β being the critical BI field strength ($\beta = 1/2\pi\alpha'$ in string theory). In four dimensions this is equivalent to

$$L = \frac{\beta^2}{4\pi} (\mathcal{R} - 1), \quad \mathcal{R} = \sqrt{1 + \frac{F_{\mu\nu}F^{\mu\nu}}{\beta^2} - \frac{(\tilde{F}_{\mu\nu}F^{\mu\nu})^2}{16\beta^4}}. \quad (58)$$

In the non-Abelian case the strength tensor $F^{\mu\nu}$ is matrix valued, and the prescription (Tseytlin [25]) is more complicated: the symmetrized trace, which is calculated expanding the Lagrangian in powers of $F_a^{\mu\nu}T^a$ (T^a being the gauge group generators), then symmetrizing all products of T^a involved and only afterwards taking the trace. Symbolically this is given by the expression

$$L_{Str} = \frac{\beta^2}{4\pi} \text{Str} \left(\sqrt{-\det(g_{\mu\nu} + F_{\mu\nu}/\beta)} - \sqrt{-g} \right), \quad (59)$$

but actually this is a useful form if one is able to perform a subsequent resummation. Fortunately, this is possible in the closed form for the homogeneous and isotropic SU(2) YM field and the metric $ds^2 = N^2 dt^2 - a^2 dl_3^2$ leading to [27]:

$$L_{Str} = -Na^3 \frac{1 - 2K^2 + 2V^2 - 3V^2K^2}{\sqrt{1 - K^2 + V^2 - K^2V^2}}, \quad K^2 = \frac{\dot{w}^2}{\beta^2 a^2 N^2}, \quad V^2 = \frac{(w^2 - k)^2}{\beta^2 a^4}. \quad (60)$$

A simpler (the ordinary trace) prescription for the NBI Lagrangian consists in summation over color indices in the field invariants $F_{\mu\nu}^a F^{a\mu\nu}$, $\tilde{F}_{\mu\nu}^a F^{a\mu\nu}$ in the square root form of the Lagrangian (58). This gives

$$L = -Na^3 \sqrt{1 - 3K^2 + 3V^2 - 9K^2V^2}. \quad (61)$$

A. NBI cosmology

Homogeneous and isotropic NBI cosmology with an ordinary trace Lagrangian turns out to be completely solvable by separation of variables [26]. It leads to an interesting equation of state:

$$p = \frac{\varepsilon(\varepsilon_c - \varepsilon)}{3(\varepsilon_c + \varepsilon)}, \quad (62)$$

where $\varepsilon_c = \beta/4\pi$ is the critical energy density, corresponding to vanishing pressure. For larger energies the pressure becomes negative, its limiting value being

$$p = -\varepsilon/3. \quad (63)$$

This is the equation of state of an ensemble of non-interacting isotropically distributed straight Nambu-Goto strings (which indicates on the stringy origin of the NBI Lagrangian). In the low-energy limit the YM equation of state $p = \varepsilon/3$ is recovered. Thus, the NBI FRW cosmology smoothly interpolates between the string gas cosmology and the hot Universe. The energy density is

$$\varepsilon = \varepsilon_c \left(\sqrt{\frac{a^4 + 3(w^2 - k)^2}{a^4 - 3\dot{w}^4}} - 1 \right). \quad (64)$$

From the YM (NBI) equation one obtains the following evolution equation for the energy density:

$$\dot{\varepsilon} = -2 \frac{\dot{a}}{a} \frac{\varepsilon(\varepsilon + 2\varepsilon_c)}{\varepsilon + \varepsilon_c}, \quad (65)$$

which can be integrated to give

$$a^4(\varepsilon + 2\varepsilon_c)\varepsilon = \text{const.} \quad (66)$$

From this relation one can see that the behavior of the NBI field interpolates between two patterns: 1) for large energy densities ($\varepsilon \gg \varepsilon_c$) the energy density scales as $\varepsilon \sim a^{-2}$; 2) for small densities $\varepsilon \ll \varepsilon_c$ one has a radiation law $\varepsilon \sim a^{-4}$.

Remarkably, the equation for the scale factor a can be decoupled ($g = \beta G$):

$$\ddot{a} = -\frac{2ga(\dot{a}^2 + k)}{2ga^2 + 3(\dot{a}^2 + k)} \quad (67)$$

and admits the first integral

$$3(\dot{a}^2 + k)^2 + 4ga^2(\dot{a}^2 + k) = C, \quad (68)$$

which allows to draw phase portraits for different k

- **Closed.** The only singular point is $a = 0, b = 0$, ($b = \dot{a}$) which is a center with the eigenvalues $\pm i\sqrt{6g}$. Solutions evolve from left to right in the upper half-plane as time changes from $-\infty$ to ∞ , and from right to left in the lower half-plane. All solutions are of an oscillating type: they start at the singularity ($a = 0$) and after a stage of expansion shrink to another singularity. The global qualitative behavior of solutions does not differ substantially from that in the conformally invariant YM field model, except near the singularity:

$$a(t) = b_0 t - \frac{b_0 g}{9} t^3 + O(t^5), \quad (69)$$

where b_0 is a free parameter. Absence of the quadratic term means that the Universe starts with zero acceleration in accord with the equation of state $p \approx -\varepsilon/3$ at high densities.

- **Spatially flat.** There is a singular line $b = 0$ each point of which represents a solution for an empty space (Minkowski spacetime). This set is degenerate, and there are no solutions that reach this curve for finite values of a . All solutions in the upper half-plane after initial singularity expand infinitely. A remarkable fact is that for this case one can write an exact solution for a in an implicit form:

$$4\sqrt{g}(t - t_0) = \sqrt{3}(\Omega - \arctan \Omega^{-1} + \pi/2), \quad (70)$$

where $\Omega = \sqrt{2}a/\sqrt{\sqrt{a^4 + C} - a^2}$. The metric singularity is reached at $t = t_0$.

- **Open.** Physical domain is $b < -1, b > 1$. There is a center at $a = 0, b = 0$ with the eigenvalues $\pm i\sqrt{6g}$, but it lies outside the boundary of the physical region. Other singular points are ($a = 0, b = \pm 1$). These points are degenerate and cannot be reached from any point lying in the physically allowed domain of the phase plane. The only solutions which start from them are the separatrices $b = \pm 1$ that represent (part of) the flat Minkowski spacetime in special coordinates. One can easily see that all solutions in the upper part of the physical domain $\dot{a} > 1$ start from the singularity and then move to $a \rightarrow \infty, \dot{a} \rightarrow 1$.

B. NBI on the brane

Replacement of the standard YM Lagrangian by the Born-Infeld one breaks conformal symmetry, providing deviation from the hot equation of state and creating negative pressure. Surprisingly enough, putting the same NBI theory into the RS2 framework gives rise to an exact restoration of the conformal symmetry by the brane non-linear corrections [27]. Choosing the ordinary trace action

$$S = \lambda \text{Tr} \int \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu}/\beta)} d^4x - \kappa^2 \int (R_5 + 2\Lambda_5) \sqrt{-g_5} d^5x, \quad (71)$$

where the brane tension λ plays a role of the BI critical energy density, one obtains the constraint equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{6} \Lambda + \frac{\kappa^4}{36} (\lambda + \varepsilon)^2 + \frac{\mathcal{E}}{a^4} - \frac{k}{a^2}, \quad (72)$$

where \mathcal{E} is the integration constant corresponding to the bulk Weyl tensor projection (“dark radiation”) and, as usual,

$$\Lambda_4 = \frac{1}{2} \kappa^2 (\Lambda + \frac{1}{6} \kappa^2 \lambda^2), \quad G_{(4)} = \frac{\kappa^4 \lambda}{48\pi}. \quad (73)$$

The energy density in this model scales as

$$\varepsilon = \lambda \left(\sqrt{1 + C/a^4} - 1 \right), \quad (74)$$

where C is the integration constant. Surprisingly, the constraint equation comes back to that of the YM conformally symmetric cosmology

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{(4)}}{3} \Lambda_4 + \frac{\mathcal{C}}{a^4} - \frac{k}{a^2}, \quad (75)$$

where the constant $\mathcal{C} = \mathcal{E} + \kappa^4 \lambda^2 C/36$ includes contributions from both the “dark radiation” and the YM energy density.

V. CONFORMAL SYMMETRY BREAKING AND DARK ENERGY

Conformal symmetry breaking in NBI theory demonstrates the occurrence of the negative pressure, but its extremal value $p = -\varepsilon/3$ is still insufficient for DE. Meanwhile, a stronger violation of conformal symmetry may provide an equation of state with $\varepsilon \sim -1$. Such violation can be of different nature:

- Quantum corrections,
- Non-minimal coupling to gravity,
- Dilaton and other coupled scalar fields including Higgs,
- String theory corrections.

Here we just explore some model Lagrangians to see the necessary conditions for DE. Assuming the Lagrangian to be an arbitrary function $L(\mathcal{F}, \mathcal{G})$ of invariants

$$\mathcal{F} = -F_{\mu\nu}^a F^{a\mu\nu}/2 \quad \text{and} \quad \mathcal{G} = -\tilde{F}_{\mu\nu}^a F^{a\mu\nu}/4, \quad (76)$$

one finds for the pressure and the energy density (conformal time):

$$\begin{aligned} p &= L + \left(2 \frac{\partial L}{\partial \mathcal{F}} [2(k - w^2)^2 - \dot{w}^2] - 3 \frac{\partial L}{\partial \mathcal{G}} \dot{w}(k - w^2) \right) a^{-4}, \\ \varepsilon &= -L + \left(6 \frac{\partial L}{\partial \mathcal{F}} \dot{w}^2 + 3 \frac{\partial L}{\partial \mathcal{G}} \dot{w}(k - w^2) \right) a^{-4}. \end{aligned} \quad (77)$$

For a simple estimate consider the power-low dependence:

$$L \sim \mathcal{F}(\mathcal{F}/\mu^2)^{\nu-1}, \quad (78)$$

where μ has the dimension of mass. Then in the electric (kinetic) dominance regime one obtains

$$W = \frac{p}{\varepsilon} = \frac{3-2\nu}{3(2\nu-1)}. \quad (79)$$

For certain ν this quantity may be arbitrarily close to $W = -1$ or even less. An electric phantom regime is thus possible.

In the magnetic (potential) dominance regime one obtains

$$W = 4\nu/3 - 1. \quad (80)$$

The value $W = -1$ can not be reached, but an admissible DE regime is also possible. These regimes are transient since during the evolution the electric part transforms to the magnetic and vice versa.

Another plausible form of the lagrangian (suggested by quantum corrections) is logarithmic [8]:

$$L \sim \mathcal{F} \ln(\mathcal{F}/\mu^2). \quad (81)$$

Then the energy density is

$$\varepsilon = 3 \left(T[\ln(\mathcal{F}/\mu^2) + 2] + V \ln(\mathcal{F}/\mu^2) \right) a^{-4}, \quad (82)$$

and the equation of state is

$$W = \frac{p}{\varepsilon} = \frac{(T+V) \ln(\mathcal{F}/\mu^2) + 2(2V-T)}{3(T+V) \ln(\mathcal{F}/\mu^2) + 6T}, \quad (83)$$

where $T = \dot{w}^2$, $V = (k - w^2)^2$. It is easy to see that $W \sim -1$ for $\ln(\mathcal{F}/\mu^2) \sim -1$. In this case the DE regime is transient. The phantom regime is also possible.

Thus, the DE conditions for the homogeneous and isotropic YM field can arise indeed as a result of sufficiently strong breaking of the conformal symmetry.

VI. MISCELLANIES AND OUTLOOK

Though not directly related, we would like to mention here an interesting attempt to derive dark energy scale cosmological constant from gluon vacuum of QCD by Klinkhamer and Volovik [30]. The issues related to chaotic behavior of homogeneous non-isotropic YM configurations were discussed in [31]. Possible applications of YM fields to cosmology up to partial identification of CMB with “cold” YM matter as discussed in the Sec. 2 were recently proposed by Tipler [32]. We want, however, to conclude with more conservative viewpoint that further work is needed to understand whether YM fields can be relevant in realistic cosmology indeed. First of all this includes deeper investigation of the coupled YMH dynamics and a thorough analysis of cosmological perturbations in various EYM scenarios.

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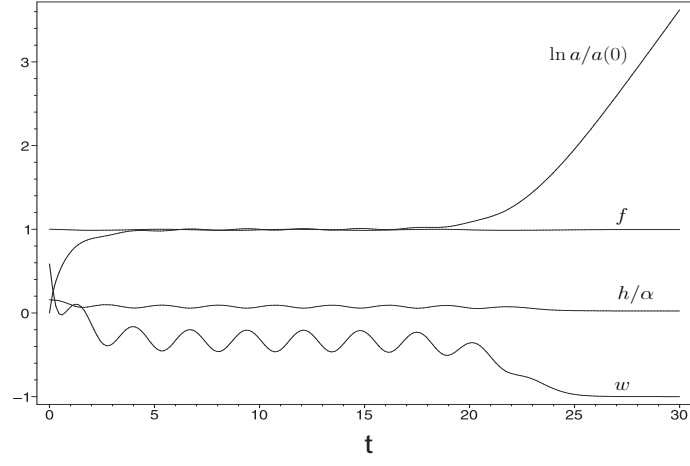


Рис. 1: The amplitudes $h(t)/\alpha$, $f(t)$ of Higgs and YM fields and the state parameter $w(t) = p/\varepsilon$ for a typical solution with long quasistationary state.

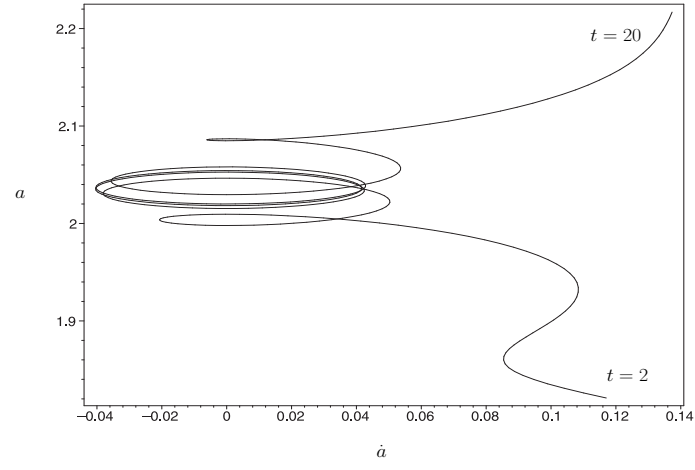


Рис. 2: Phase trajectory for the scale factor in vicinity of the stationary state.

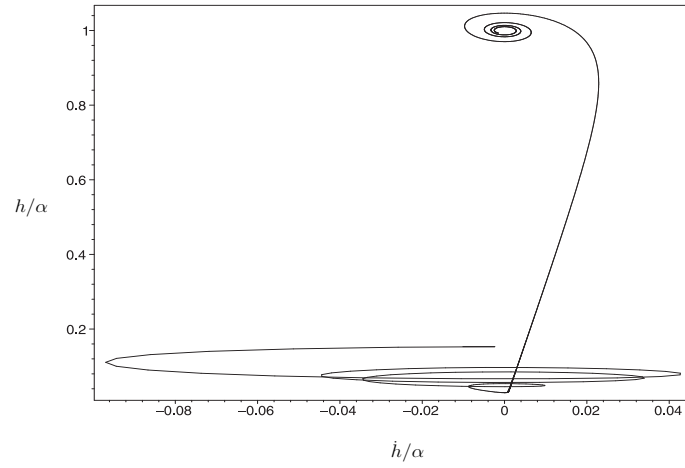


Рис. 3: Phase trajectory of the Higgs field demonstrating the jump of the minimum of the scalar field potential from $h_{min} \simeq 0$ (equals to zero when $j = 0$) to α with the growing scale factor.

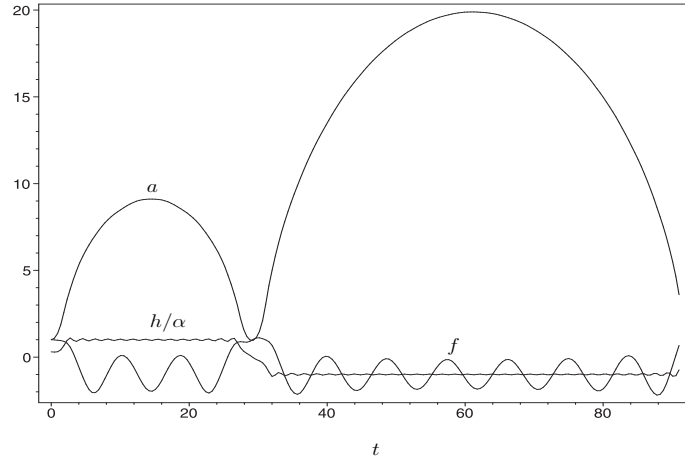


Рис. 4: The field amplitudes and the scale factor for two subsequent cycles with different parameters.

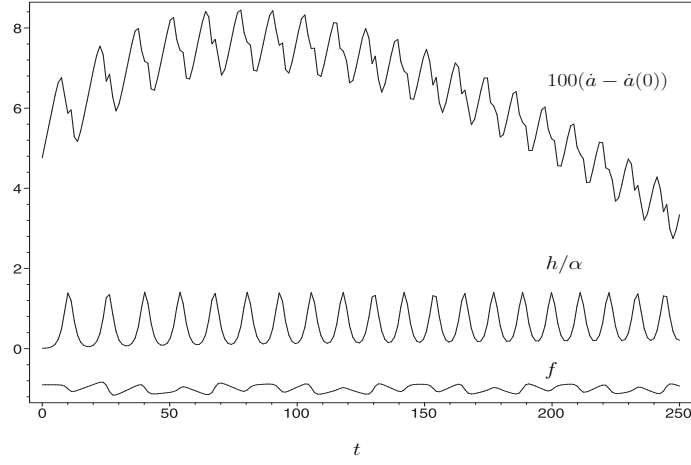


Рис. 5: A typical solution for small α (on this plot $\alpha = 10^{-6}$). Initially the scalar field amplitude is large enough to ensure cosmic acceleration. With decreasing amplitude an accelerated expansion changes to contraction.

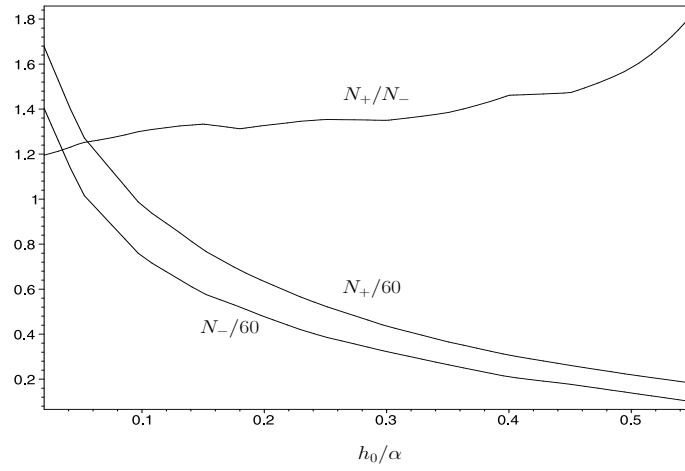


Рис. 6: The number of e -folds during inflation in the case of simple slow roll inflation, N_- , and hybrid inflation, N_+ , as functions of initial value of the scalar field, h_0 .

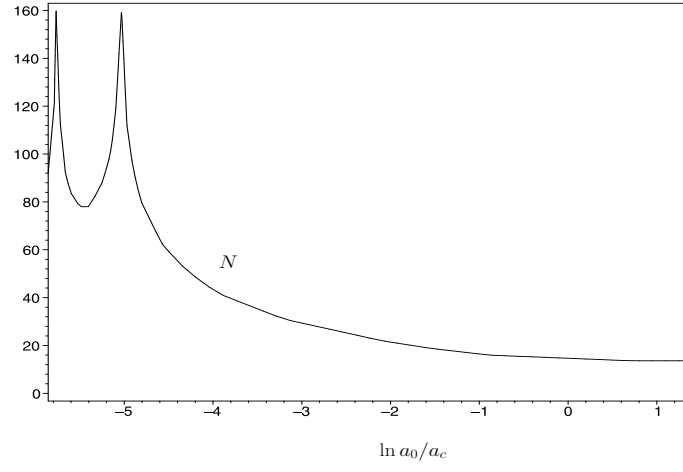


Рис. 7: The dependence of the number N of e -folds on the initial value of the scale factor a_0 , as compared with the critical value $a_c \simeq \sqrt{6}/(\alpha\beta)$. When a_0 reaches a_c , the hybrid inflation reduces to the simple slow roll inflation. The peaks in the left part of the plot describe the divergence of N due to an infinite inflation, when the Higgs field is driven exactly to the top of the Higgs potential.